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Renormalization scheme dependence of the two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM

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Abstract Reaching a theoretical accuracy in the prediction of the lightest MSSM Higgs-boson mass, M_h, at the level of the current experimental precision requires the inclusion of momentum-dependent contributions at the two-loop level. Recently two groups presented the two-loop QCD momentum-dependent corrections to M_h (Borowka et al., Eur Phys J C 74(8):2994, 2014; Degrassi et al., Eur Phys J C 75(2):61, 2015), using a hybrid on-shell-$\overline{\rm DR}$ scheme, with apparently different results. We show that the differences can be traced back to a different renormalization of the top-quark mass, and that the claim in Ref. Degrassi et al. (Eur Phys J C 75(2):61, 2015) of an inconsistency in Ref. Borowka et al. (Eur Phys J C 74(8):2994, 2014) is incorrect. We furthermore compare consistently the results for M_h obtained with the top-quark mass renormalized on-shell and $\overline{\rm DR}$. The latter calculation has been added to the `FeynHiggs` package and can be used to estimate missing higher-order corrections beyond the two-loop level.

1 Introduction

The particle discovered in the Higgs-boson searches by ATLAS [3] and CMS [4] at CERN shows, within experimental and theoretical uncertainties, properties compatible with the Higgs boson of the Standard Model (SM) [5–7]. It can also be interpreted as the Higgs boson of extended models, however, where the lightest Higgs boson of the Minimal Supersymmetric Standard Model (MSSM) [8–10] is a prime candidate.

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The Higgs sector of the MSSM with two scalar doublets accommodates five physical Higgs bosons. In lowest order these are the light and heavy $\mathcal{CP}$-even h and H, the $\mathcal{CP}$-odd A, and the charged Higgs bosons $H^\pm$. At tree level, the Higgs sector can be parameterized in terms of the gauge couplings, the mass of the $\mathcal{CP}$-odd Higgs boson, M_A, and $\tan\beta \equiv v_2/v_1$, the ratio of the two vacuum expectation values; all other masses and mixing angles follow as predictions.

Higher-order contributions can give large corrections to the tree-level relations [11–13], and in particular to the mass of the lightest Higgs boson, M_h. For the MSSM[1] with real parameters the status of higher-order corrections to the masses and mixing angles in the neutral Higgs sector is quite advanced; see Refs. [19–26] for the calculations of the full one-loop level. At the two-loop level [18,27–44] in particular the $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ contributions ($\alpha_t \equiv h_t^2/(4\pi)$, h_t being the top-quark Yukawa coupling) to the self-energies – evaluated in the Feynman-diagrammatic (FD) as well as in the effective potential (EP) method – as well as the $\mathcal{O}(\alpha_b\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_b)$ and $\mathcal{O}(\alpha_b^2)$ contributions – evaluated in the EP approach – are known for vanishing external momenta. An evaluation of the momentum dependence at the two-loop level in a pure $\overline{\rm DR}$ calculation was presented in Ref. [45]. The latest status of the momentum-dependent two-loop corrections will be discussed below. A (nearly) full two-loop EP calculation, including even the leading three-loop corrections, has also been published [46–50,52,53]. Within the EP method all contributions are evaluated at zero external momentum, however, in contrast to the FD method which in principle allows for non-vanishing external momenta. Furthermore, the calculation presented in Refs. [46–50,52,53] is not publicly available as a computer code for Higgs-boson mass calculations. Subsequently, another leading three-loop

[1] We concentrate here on the case with real parameters. For the case of complex parameters see Refs. [14–18] and references therein.

calculation of $\mathcal{O}(\alpha_t\alpha_s^2)$, depending on the various SUSY mass hierarchies, was completed [54–56], resulting in the code `H3m` which adds the three-loop corrections to the `FeynHiggs` [14,29,57–60] result. Most recently, a combination of the full one-loop result, supplemented with leading and subleading two-loop corrections evaluated in the FD/EP method and a resummation of the leading and subleading logarithmic corrections from the scalar-top sector has been published [60] in the latest version of the code `FeynHiggs`.

The measured mass value of the observed Higgs boson is currently known to about 250 MeV accuracy [5], reaching the level of a precision observable. At a future linear collider (ILC), the precise determination of the light Higgs-boson properties and/or heavier MSSM Higgs bosons within the kinematic reach will be possible [61]. In particular, a mass measurement of the light Higgs boson with an accuracy below ~ 0.05 GeV is anticipated [62].

In Ref. [59] the remaining theoretical uncertainty in the calculation of M_h, from unknown higher-order corrections, was estimated to be up to 3 GeV, depending on the parameter region; see also Refs. [60,63] for updated results. As the accuracy of the M_h prediction should at least match the one of the experimental result, higher-order corrections which do not dominate the size of the Higgs-boson mass values have to be included in the Higgs-boson mass predictions.

To better control the size of momentum-dependent contributions, we recently presented the calculation of the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections to M_h (the leading momentum-dependent two-loop QCD corrections). The calculation was performed in a hybrid on-shell/$\overline{\text{DR}}$ scheme [1] at the two-loop level, where M_A and the tadpoles are renormalized on-shell (OS), whereas the Higgs-boson fields and $\tan\beta$ are renormalized $\overline{\text{DR}}$. At the one-loop level the top/stop parameters are renormalized OS.[2] Subsequently, in Ref. [2] this calculation was repeated with a different result (also, a calculation in a pure $\overline{\text{DR}}$ scheme as well as the two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ were presented). Within Ref. [2] the discrepancy between Refs. [1,2] was explained by an inconsistency in the renormalization scheme used for the Higgs-boson field renormalization in Ref. [1].

In this paper we demonstrate that this claim is incorrect. The renormalization scheme for the Higgs-boson fields used in Ref. [1] is (up to corrections beyond the two-loop level) identical to the one employed in Ref. [2]. We clarify that the differences between the two results originates in a difference of the top-quark-mass renormalization scheme. While in Ref. [1] a full OS renormalization was used, in Ref. [2] the contributions to the top-quark self-energy of $\mathcal{O}(\varepsilon)$ (with $4 - D = 2\varepsilon$, D being the space-time dimension) were neglected, leading to the observed numerical differences. We also demonstrate how this difference in the treatment of the contributions from the top-quark mass can be linked to a difference in the two-loop field renormalization constant and explain why this difference should be regarded as a theoretical uncertainty at the two-loop level, which would be fixed only at three-loop order.

We further present a consistent calculation of the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections to M_h in a scheme where the top quark is renormalized $\overline{\text{DR}}$, whereas the scalar tops continue to be renormalized OS. This new scheme is available from `FeynHiggs` version 2.11.1 on, allowing for an improved estimate of (some) unknown higher-order corrections beyond the two-loop level originating from the top/stop sector.

The paper is organized as follows. An overview of the relevant sectors and the renormalization employed in our calculation is given in Sect. 2. In Sect. 3 we compare analytically and numerically the results of Refs. [1,2]. Results obtained using the $\overline{\text{DR}}$ scheme for the top-quark mass are given in Sect. 4. Our conclusions are given in Sect. 5.

2 The relevant sectors and their renormalization

2.1 The Higgs-boson sector of the MSSM

The MSSM requires two scalar doublets, which are conventionally written in terms of their components as follows:

$$\mathcal{H}_1 = \begin{pmatrix} \mathcal{H}_1^0 \\ \mathcal{H}_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - i\chi_1^0) \\ -\phi_1^- \end{pmatrix},$$
$$\mathcal{H}_2 = \begin{pmatrix} \mathcal{H}_2^+ \\ \mathcal{H}_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\chi_2^0) \end{pmatrix}.$$

The bilinear part of the Higgs potential leads to the tree-level mass matrix for the neutral $\mathcal{CP}$-even Higgs bosons,

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} = \begin{pmatrix} M_A^2\sin^2\beta + M_Z^2\cos^2\beta & -(M_A^2+M_Z^2)\sin\beta\cos\beta \\ -(M_A^2+M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}, \tag{1}$$

in the (ϕ_1, ϕ_2) basis, expressed in terms of the Z boson mass, M_Z, M_A, and the angle β. Diagonalization via the angle α yields the tree-level masses $m_{h,\text{tree}}$ and $m_{H,\text{tree}}$. Below we also use M_W, denoting the W boson mass and s_w, the sine of the weak mixing angle, $s_\text{w} = \sqrt{1 - c_\text{w}^2} = \sqrt{1 - M_W^2/M_Z^2}$.

The higher-order-corrected $\mathcal{CP}$-even Higgs-boson masses in the MSSM are obtained from the corresponding propagators dressed by their self-energies. The calculation of these and their renormalization is performed in the (ϕ_1, ϕ_2) basis,

[2] From a technical point of view we calculated the momentum-dependent two-loop self-energy diagrams numerically using the program `SecDec` [64–66].

which has the advantage that the mixing angle α does not appear and expressions are in general simpler. The inverse propagator matrix in the (ϕ_1, ϕ_2) basis is given by

$$(\Delta_{\rm Higgs})^{-1} = -{\rm i}\begin{pmatrix} p^2 - m_{\phi_1}^2 + \hat{\Sigma}_{\phi_1}(p^2) & -m_{\phi_1\phi_2}^2 + \hat{\Sigma}_{\phi_1\phi_2}(p^2) \\ -m_{\phi_1\phi_2}^2 + \hat{\Sigma}_{\phi_1\phi_2}(p^2) & p^2 - m_{\phi_2}^2 + \hat{\Sigma}_{\phi_2}(p^2) \end{pmatrix}, \tag{2}$$

where $\hat{\Sigma}(p^2)$ denote the renormalized Higgs-boson self-energies, p being the external momentum. The renormalized self-energies can be expressed through the unrenormalized self-energies, $\Sigma(p^2)$, and counterterms involving renormalization constants δm^2 and δZ from parameter and field renormalization. With the self-energies expanded up to two-loop order, $\hat{\Sigma} = \hat{\Sigma}^{(1)} + \hat{\Sigma}^{(2)}$, one has for the $\mathcal{CP}$-even part at the i-loop level ($i = 1, 2$),

$$\hat{\Sigma}_{\phi_1}^{(i)}(p^2) = \Sigma_{\phi_1}^{(i)}(p^2) + \delta Z_{\phi_1}^{(i)}(p^2 - m_{\phi_1}^2) - \delta m_{\phi_1}^{2(i)}, \tag{3a}$$
$$\hat{\Sigma}_{\phi_1\phi_2}^{(i)}(p^2) = \Sigma_{\phi_1\phi_2}^{(i)}(p^2) - \delta Z_{\phi_1\phi_2}^{(i)}\, m_{\phi_1\phi_2}^2 - \delta m_{\phi_1\phi_2}^{2(i)}, \tag{3b}$$
$$\hat{\Sigma}_{\phi_2}^{(i)}(p^2) = \Sigma_{\phi_2}^{(i)}(p^2) + \delta Z_{\phi_2}^{(i)}(p^2 - m_{\phi_2}^2) - \delta m_{\phi_2}^{2(i)}. \tag{3c}$$

At the two-loop level the expressions in Eqs. (3) do not contain contributions of the type (1-loop) × (1-loop); such terms do not appear at $\mathcal{O}(\alpha_t\alpha_s)$ and hence can be omitted in the context of this paper. For the general expressions see Ref. [18].

Beyond the one-loop level, unrenormalized self-energies contain sub-loop renormalizations. At the two-loop level, these are one-loop diagrams with counterterm insertions at the one-loop level.

2.2 Renormalization

The following section summarizes the renormalization worked out in Ref. [1], based on Ref. [29]. The field renormalization is carried out by assigning one renormalization constant to each doublet,

$$\mathcal{H}_1 \to (1 + \tfrac{1}{2}\delta Z_{\mathcal{H}_1})\,\mathcal{H}_1, \quad \mathcal{H}_2 \to (1 + \tfrac{1}{2}\delta Z_{\mathcal{H}_2})\mathcal{H}_2, \tag{4}$$

which can be expanded to one- and two-loop order according to

$$\delta Z_{\mathcal{H}_1} = \delta Z_{\mathcal{H}_1}^{(1)} + \delta Z_{\mathcal{H}_1}^{(2)}, \quad \delta Z_{\mathcal{H}_2} = \delta Z_{\mathcal{H}_2}^{(1)} + \delta Z_{\mathcal{H}_2}^{(2)}. \tag{5}$$

The field renormalization constants appearing in (3) are then given by

$$\delta Z_{\phi_1}^{(i)} = \delta Z_{\mathcal{H}_1}^{(i)}, \quad \delta Z_{\phi_2}^{(i)} = \delta Z_{\mathcal{H}_2}^{(i)},$$
$$\delta Z_{\phi_1\phi_2}^{(i)} = \tfrac{1}{2}(\delta Z_{\mathcal{H}_1}^{(i)} + \delta Z_{\mathcal{H}_2}^{(i)}). \tag{6}$$

The mass counterterms $\delta m_{ab}^{2(i)}$ in Eq. (3) are derived from the Higgs potential, including the tadpoles, by the following parameter renormalization:

$$M_A^2 \to M_A^2 + \delta M_A^{2(1)} + \delta M_A^{2(2)},$$
$$T_1 \to T_1 + \delta T_1^{(1)} + \delta T_1^{(2)},$$
$$M_Z^2 \to M_Z^2 + \delta M_Z^{2(1)} + \delta M_Z^{2(2)},$$
$$T_2 \to T_2 + \delta T_2^{(1)} + \delta T_2^{(2)},$$
$$\tan\beta \to \tan\beta\left(1 + \delta\tan\beta^{(1)} + \delta\tan\beta^{(2)}\right). \tag{7}$$

The parameters T_1 and T_2 are the terms linear in ϕ_1 and ϕ_2 in the Higgs potential. The renormalization of the Z-mass M_Z does not contribute to the $\mathcal{O}(\alpha_s\alpha_t)$ corrections we are pursuing here; it is listed for completeness only.

The basic renormalization constants for parameters and fields have to be fixed by renormalization conditions according to a renormalization scheme. Here we choose the on-shell scheme for the parameters and the $\overline{\rm DR}$ scheme for field renormalization and give the expressions for the two-loop part. This is consistent with the renormalization scheme used at the one-loop level.

The tadpole coefficients are chosen to vanish at all orders; hence their two-loop counterterms follow from

$$T_{1,2}^{(2)} + \delta T_{1,2}^{(2)} = 0, \quad \text{i.e.} \quad \delta T_1^{(2)} = -T_1^{(2)}, \quad \delta T_2^{(2)} = -T_2^{(2)}, \tag{8}$$

where $T_1^{(2)}$, $T_2^{(2)}$ are obtained from the two-loop tadpole diagrams. The two-loop renormalization constant of the A-boson mass reads

$$\delta M_A^{2(2)} = {\rm Re}\,\Sigma_{AA}^{(2)}(M_A^2), \tag{9}$$

in terms of the A-boson unrenormalized self-energy Σ_{AA}. The appearance of a non-zero momentum in the self-energy goes beyond the $\mathcal{O}(\alpha_t\alpha_s)$ corrections evaluated in Refs. [27–29,35].

For the renormalization constants $\delta Z_{\mathcal{H}_1}$, $\delta Z_{\mathcal{H}_2}$, and $\delta\tan\beta$ several choices are possible; see the discussion in [67–69]. As shown there, the most convenient choice is a $\overline{\rm DR}$ renormalization of $\delta\tan\beta$, $\delta Z_{\mathcal{H}_1}$, and $\delta Z_{\mathcal{H}_2}$, which at the two-loop level reads

$$\delta Z_{\mathcal{H}_1}^{(2)} = \delta Z_{\mathcal{H}_1}^{\overline{\rm DR}(2)} = -\left[{\rm Re}\,\Sigma_{\phi_1}^{\prime(2)}\right]_{|p^2=0}^{\rm div}, \tag{10a}$$
$$\delta Z_{\mathcal{H}_2}^{(2)} = \delta Z_{\mathcal{H}_2}^{\overline{\rm DR}(2)} = -\left[{\rm Re}\,\Sigma_{\phi_2}^{\prime(2)}\right]_{|p^2=0}^{\rm div}, \tag{10b}$$
$$\delta\tan\beta^{(2)} = \delta\tan\beta^{\overline{\rm DR}(2)} = \tfrac{1}{2}\left(\delta Z_{\mathcal{H}_2}^{(2)} - \delta Z_{\mathcal{H}_1}^{(2)}\right). \tag{10c}$$

The term in Eq. (10c) is in general not the proper expression beyond one-loop order even in the $\overline{\rm DR}$ scheme. For our

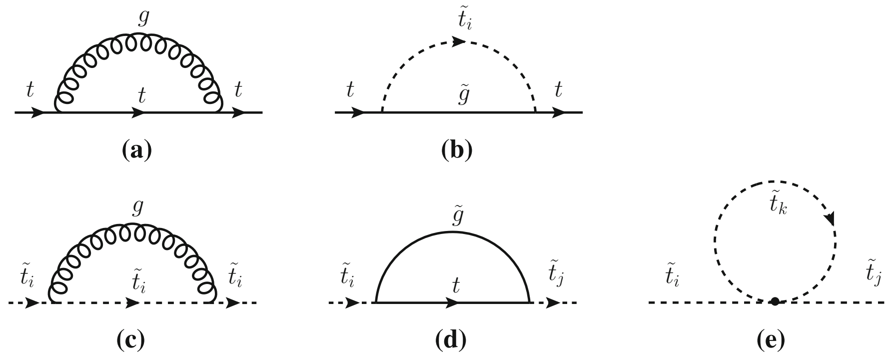

Fig. 1 Generic one-loop diagrams for subrenormalization counterterms for the top quark (*upper row*) and for the scalar tops (*lower row*) $(i, j, k = 1, 2)$

approximation, however, with only the top Yukawa coupling at the two-loop level, it is the correct $\overline{\rm DR}$ form [70,71].

The two-loop mass counterterms in the renormalized self-energies (3) are now expressed in terms of the two-loop parameter renormalization constants, determined above, as follows:

$$\begin{aligned}\delta m^{2(2)}_{\phi_1} &= \delta M_Z^{2(2)} \cos^2\beta + \delta M_A^{2(2)} \sin^2\beta \\ &\quad - \delta T_1^{(2)} \frac{e}{2M_W s_\mathrm{w}} \cos\beta(1+\sin^2\beta) \\ &\quad + \delta T_2^{(2)} \frac{e}{2M_W s_\mathrm{w}} \cos^2\beta \sin\beta \\ &\quad + 2\,\delta\tan\beta^{(2)} \cos^2\beta \sin^2\beta\,(M_A^2 - M_Z^2), \qquad (11a)\end{aligned}$$

$$\begin{aligned}\delta m^{2(2)}_{\phi_1\phi_2} &= -(\delta M_Z^{2(2)} + \delta M_A^{2(2)}) \sin\beta\cos\beta \\ &\quad - \delta T_1^{(2)} \frac{e}{2M_W s_\mathrm{w}} \sin^3\beta - \delta T_2^{(2)} \frac{e}{2M_W s_\mathrm{w}} \cos^3\beta \\ &\quad - \delta\tan\beta^{(2)} \cos\beta\sin\beta\cos 2\beta\,(M_A^2 + M_Z^2), \qquad (11b)\end{aligned}$$

$$\begin{aligned}\delta m^{2(2)}_{\phi_2} &= \delta M_Z^{2(2)} \sin^2\beta + \delta M_A^{2(2)} \cos^2\beta \\ &\quad + \delta T_1^{(2)} \frac{e}{2M_W s_\mathrm{w}} \sin^2\beta\cos\beta \\ &\quad - \delta T_2^{(2)} \frac{e}{2M_W s_\mathrm{w}} \sin\beta(1+\cos^2\beta) \\ &\quad - 2\,\delta\tan\beta^{(2)} \cos^2\beta\sin^2\beta\,(M_A^2 - M_Z^2). \qquad (11c)\end{aligned}$$

The Z-mass counterterm is again kept for completeness; it does not contribute in the approximation of $\mathcal{O}(\alpha_t\alpha_s)$ considered here.

2.3 Diagram evaluation

Our calculation is performed in the Feynman-diagrammatic (FD) approach. To arrive at expressions for the unrenormalized self-energies and tadpoles at $\mathcal{O}(\alpha_t\alpha_s)$, the evaluation of genuine two-loop diagrams and one-loop graphs with counterterm insertions is required. For the counterterm insertions, described in Sect. 2.4, one-loop diagrams with external top quarks/squarks have to be evaluated as well, as displayed in Fig. 1. The calculation is performed in dimensional reduction [72,73].

The complete set of contributing Feynman diagrams was generated with the program `FeynArts` [74–76] (using the model file including counterterms from Ref. [77]), tensor reduction and the evaluation of traces was done with support from the programs `FormCalc` [78] and `TwoCalc` [79,80], yielding algebraic expressions in terms of the scalar one-loop functions A_0, B_0 [81], the massive vacuum two-loop functions [82], and two-loop integrals which depend on the external momentum. These integrals were evaluated with the program `SecDec` [64–66], where up to four different masses in 34 different mass configurations needed to be considered, with differences in the kinematic invariants of several orders of magnitude.

2.4 The scalar-top sector of the MSSM

The bilinear part of the top-squark Lagrangian,

$$\mathcal{L}_{\tilde{t},\mathrm{mass}} = -\begin{pmatrix}\tilde{t}_L^\dagger, \tilde{t}_R^\dagger\end{pmatrix} \mathbf{M}_{\tilde{t}} \begin{pmatrix}\tilde{t}_L \\ \tilde{t}_R\end{pmatrix}, \qquad (12)$$

contains the stop-mass matrix

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + M_Z^2\cos 2\beta\,(T_t^3 - Q_t s_\mathrm{w}^2) & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + M_Z^2\cos 2\beta\, Q_t\, s_\mathrm{w}^2 \end{pmatrix}, \qquad (13)$$

with

$$X_t = A_t - \mu\,\cot\beta \qquad (14)$$

where Q_t and T_t^3 denote the charge and isospin of the top quark, A_t the trilinear coupling between the Higgs bosons and the scalar tops, and μ the Higgsino mass parameter. Below we use $M_{\rm SUSY} := M_{\tilde{t}_L} = M_{\tilde{t}_R}$ for our numerical evaluation. The analytical calculation was performed for arbitrary $M_{\tilde{t}_L}$ and $M_{\tilde{t}_R}$, however. $\mathbf{M}_{\tilde{t}}$ can be diagonalized with the help of a unitary transformation matrix $\mathbf{U}_{\tilde{t}}$, parameterized by a mixing angle $\theta_{\tilde{t}}$, to provide the eigenvalues $m^2_{\tilde{t}_1}$ and $m^2_{\tilde{t}_2}$ as the squares of the two on-shell top-squark masses.

For the evaluation of the $\mathcal{O}(\alpha_t\alpha_s)$ two-loop contributions to the self-energies and tadpoles of the Higgs sector, renormalization of the top/stop sector at $\mathcal{O}(\alpha_s)$ is required, giving rise to the counterterms for sub-loop renormalization.

We follow the renormalization at the one-loop level given in Refs. [31,83–85], where details can be found. In particular, in the context of this paper, an OS renormalization is performed for the top-quark mass as well as for the scalar-top masses. This is different from the approach pursued, for example, in Ref. [45], where a $\overline{\rm DR}$ renormalization was employed, or similarly in the pure $\overline{\rm DR}$ renormalization presented in Ref. [2]. Using the OS scheme allows us to consistently combine our new correction terms with the hitherto available self-energies included in `FeynHiggs`.

Besides employing a pure OS renormalization for the top/stop masses in our calculation, we also obtain a result in which the top-quark mass is renormalized $\overline{\rm DR}$. This new top-quark mass renormalization is included as a new option in the code `FeynHiggs`. The comparison of the results using the $\overline{\rm DR}$ and the OS renormalization allows one to estimate (some) missing three-loop corrections in the top/stop sector.

Finally, at $\mathcal{O}(\alpha_t\alpha_s)$, gluinos appear as virtual particles only at the two-loop level (hence, no renormalization for the gluinos is needed). The corresponding soft-breaking gluino mass parameter M_3 determines the gluino mass, $m_{\tilde{g}} = M_3$.

2.5 Evaluation and implementation in the program `FeynHiggs`

The resulting new contributions to the neutral $\mathcal{CP}$-even Higgs-boson self-energies, containing all momentum-dependent and additional constant terms, are assigned to the differences

$$\Delta\hat{\Sigma}_{ab}(p^2) = \hat{\Sigma}^{(2)}_{ab}(p^2) - \tilde{\Sigma}^{(2)}_{ab}(0), \quad ab = \{HH, hH, hh\}. \tag{15}$$

These are the new terms evaluated in Ref. [1], included in `FeynHiggs`. Note the tilde (not hat) on $\tilde{\Sigma}^{(2)}(0)$, which signifies that not only the self-energies are evaluated at zero external momentum but also the corresponding counterterms, following Refs. [27–29]. A finite shift $\Delta\hat{\Sigma}(0)$ therefore remains in the limit $p^2 \to 0$ due to $\delta M_A^{2(2)} = \mathrm{Re}\,\Sigma^{(2)}_{AA}(M_A^2)$ being computed at $p^2 = M_A^2$ in $\hat{\Sigma}^{(2)}$, but at $p^2 = 0$ in $\tilde{\Sigma}^{(2)}$; for details see Eqs. (9) and (11). For the sake of simplicity we will refer to these terms as $\mathcal{O}(p^2\alpha_t\alpha_s)$ despite the M_A^2 dependence.

3 Discussion of renormalization schemes

In this section we compare our results for the $\mathcal{O}(p^2\alpha_t\alpha_s)$ contributions to the MSSM Higgs-boson self-energies, as given in Ref. [1] to the ones presented subsequently in Ref. [2]. We first show analytically the agreement in the Higgs field renormalization in the two calculations and discuss the differences in the m_t renormalizations. We also present some numerical results in both schemes, demonstrating agreement with Ref. [2] once the $\mathcal{O}(\varepsilon)$ terms are dropped from the top-quark mass counterterm.

Using an OS renormalization for the top-quark mass, the counterterm is determined from the components of the $\mathcal{O}(\alpha_s)$ top-quark self-energy (Fig. 1) as follows:

$$\frac{\delta m_t^{\rm OS}}{m_t} = \frac{1}{2}\,\mathrm{Re}\left\{\left[\Sigma_t^L(m_t^2) + \Sigma_t^R(m_t^2)\right] + \left[\Sigma_t^{SL}(m_t^2) + \Sigma_t^{SR}(m_t^2)\right]\right\}, \tag{16}$$

where the top-quark self-energy is decomposed according to

$$\Sigma_t(p) = \not{p}\,\omega_-\Sigma_t^L(p^2) + \not{p}\,\omega_+\Sigma_t^R(p^2) + m_t\,\omega_-\Sigma_t^{SL}(p^2) + m_t\,\omega_+\Sigma_t^{SR}(p^2)\,. \tag{17}$$

with the projectors $\omega_\pm = \frac{1}{2}(\mathbb{1} \pm \gamma_5)$.

3.1 Analytical comparison

In the $\mathcal{O}(\alpha_t\alpha_s)$ calculation of the Higgs-boson self-energies the renormalization of the top-quark mass at $\mathcal{O}(\alpha_s)$ is required. The contributing diagrams are shown in the top row of Fig. 1. The top-quark mass counterterm is inserted into the sub-loop renormalization of the two-loop contributions to the Higgs-boson self-energies, where two sample diagrams are shown in Fig. 2. The left diagram contributes to the momentum-dependent two-loop self-energies, while the right one contributes only to the momentum-independent part.

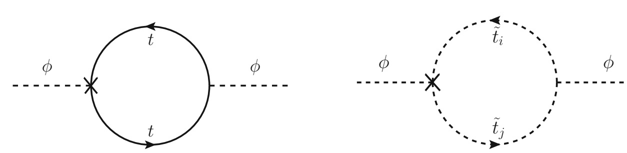

Fig. 2 One-loop subrenormalization diagram contributing to $\delta_{\Sigma_{22}}(p^2)$ and $\delta_A(p^2)$, with the counterterm insertion denoted by a *cross*. The *right* diagram only contributes to $\delta_{\Sigma_{22}}(0)$ and $\delta_A(0)$

Evaluating the expression in Eq. (16) in $4-2\varepsilon$ dimensions yields the OS top-quark mass counterterm at the one-loop level, which can be written as a Laurent expansion in ε,

$$\delta m_t^{\mathrm{OS}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}} + \delta m_t^{\mathrm{fin}} + \varepsilon\,\delta m_t^{\varepsilon} + \cdots; \tag{18}$$

higher powers in ε, indicated by the ellipses, do not contribute at the two-loop level for $\varepsilon \to 0$ after renormalization. Accordingly, the $\overline{\mathrm{DR}}$ top-quark mass counterterm is given by the singular part of Eq. (18),

$$\delta m_t^{\overline{\mathrm{DR}}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}}. \tag{19}$$

For further use we define the quantity

$$\delta m_t^{\mathrm{FIN}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}} + \delta m_t^{\mathrm{fin}}. \tag{20}$$

At $\mathcal{O}(\alpha_s)$ the OS counterterm is given as

$$\begin{aligned}\frac{\delta m_t^{\mathrm{OS}}}{m_t} = \frac{\alpha_s}{6\pi}\Bigg\{ & -2\frac{A_0(m_t^2)}{m_t^2} - 4\,B_0(m_t^2, 0, m_t^2) \\ & - 2\frac{A_0(m_{\tilde{g}}^2)}{m_t^2} + \frac{A_0(m_{\tilde{t}_1}^2)}{m_t^2} + \frac{A_0(m_{\tilde{t}_2}^2)}{m_t^2} \\ & + \frac{m_{\tilde{g}}^2 + m_t^2 - m_{\tilde{t}_1}^2 - 4\,\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,m_{\tilde{g}}\,m_t}{m_t^2} \\ & \times \mathrm{Re}\left[B_0(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_1}^2)\right] \\ & + \frac{m_{\tilde{g}}^2 + m_t^2 - m_{\tilde{t}_2}^2 + 4\,\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,m_{\tilde{g}}\,m_t}{m_t^2} \\ & \times \mathrm{Re}\left[B_0(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_2}^2)\right]\Bigg\}. \end{aligned} \tag{21}$$

The one- and two-point functions $A_0(m^2)$ and $B_0(p^2, m_1^2, m_2^2)$ are expanded in ε as follows:

$$\begin{aligned} A_0(m^2) &= \frac{1}{\varepsilon}\,A_0^{\mathrm{div}}(m^2) + A_0^{\mathrm{fin}}(m^2) + \varepsilon\,A_0^{\varepsilon}(m^2), \\ B_0(p^2, m_1^2, m_2^2) &= \frac{1}{\varepsilon}\,B_0^{\mathrm{div}}(p^2, m_1^2, m_2^2) + B_0^{\mathrm{fin}}(p^2, m_1^2, m_2^2) \\ &\quad + \varepsilon\,B_0^{\varepsilon}(p^2, m_1^2, m_2^2)\,. \end{aligned} \tag{22}$$

Consequently, the term at $\mathcal{O}(\varepsilon)$, $\delta m_t^{\varepsilon}/m_t$, is given by Eq. (21), but taking only into account the pieces $\propto A_0^{\varepsilon}$, B_0^{ε}. The special cases of $A_0^{\varepsilon}(m^2)$ and $B_0^{\varepsilon}(m^2, 0, m^2)$ are given by

$$\begin{aligned} A_0^{\varepsilon}(m^2) &= m^2\left\{1 - \log(m^2/\mu^2) + \frac{1}{2}\log^2(m^2/\mu^2) + \frac{\pi^2}{12}\right\}, \\ B_0^{\varepsilon}(m^2, 0, m^2) &= 4 - 2\,\log(m^2/\mu^2) + \frac{1}{2}\log^2(m^2/\mu^2) + \frac{\pi^2}{12}, \end{aligned} \tag{23}$$

where the factor $4\pi\,\mathrm{e}^{-\gamma_E}$ is absorbed into the renormalization scale. The expression for B_0^{ε} depending on three mass scales can be found e.g. in Ref. [86].

In our calculation in Ref. [1] we include terms up to $\mathcal{O}(\varepsilon)$, originating from the top-quark self-energy, in the top-mass counterterm,[3] i.e.

$$\delta m_t^{[1]} = \delta m_t^{\mathrm{OS}}. \tag{24}$$

The derivation in Ref. [2] proceeds differently. The renormalized Higgs-boson self-energies are first calculated in a pure $\overline{\mathrm{DR}}$ scheme. This concerns the top mass, the scalar-top masses, the Higgs field renormalization, and $\tan\beta$. In this way it is ensured that in particular the Higgs fields are renormalized using $\overline{\mathrm{DR}}$, $\delta Z_{\mathcal{H}_i} = \delta Z_{\mathcal{H}_i}^{\overline{\mathrm{DR}}}$, where this quantity contains the contribution from the one- and two-loop level. Using this pure $\overline{\mathrm{DR}}$ scheme a finite result is obtained in which all poles in $1/\varepsilon$ and $1/\varepsilon^2$ cancel, such that the limit $\varepsilon \to 0$ can be taken. Subsequently, the $\overline{\mathrm{DR}}$ top-quark mass counterterm, $\delta m_t^{\overline{\mathrm{DR}}}$, is replaced by an on-shell counterterm, and the top-quark mass definition is changed accordingly. The same procedure is applied for the scalar-top masses. Since these finite expressions for the renormalized Higgs-boson self-energies do not contain any term of $\mathcal{O}(1/\varepsilon)$, the δm_t^{ε} part of the OS top-quark mass counterterm does not contribute, i.e.

$$\delta m_t^{[2]} = \delta m_t^{\mathrm{FIN}}. \tag{25}$$

The numerical results for the renormalized Higgs-boson self-energies obtained this way differ significantly from the ones obtained in Ref. [1], as pointed out in Ref. [2].

In the following we discuss the different Higgs-boson field renormalizations, where we use the notation of $\delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{X}}}$ for the field renormalization derived using δm_t^{X}, with $\mathrm{X} = \overline{\mathrm{DR}}$, FIN, OS. The field renormalization can be decomposed into one-loop, two-loop, …parts as

$$\delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{X}}} = \delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{X}}(1)} + \delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{X}}(2)} + \cdots \tag{26}$$

In Ref. [2] it was claimed that using an OS top-quark mass renormalization from the start results in a non-$\overline{\mathrm{DR}}$ renormalization of $\delta Z_{\mathcal{H}_2}$. While it is correct that an OS value for m_t yields different results in the one- and two-loop part,

$$\delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{OS}}(1)} \neq \delta Z_{\mathcal{H}_2}^{\delta m_t^{\overline{\mathrm{DR}}}(1)}, \quad \delta Z_{\mathcal{H}_2}^{\delta m_t^{\mathrm{OS}}(2)} \neq \delta Z_{\mathcal{H}_2}^{\delta m_t^{\overline{\mathrm{DR}}}(2)}, \tag{27}$$

the sum of the one- and two-loop parts are identical, independently of the choice of the top-quark mass renormalization

[3] Taking $\mathcal{O}(\varepsilon)$ terms into account in the expressions for on-shell counterterms beyond one loop is widely used in the literature; see e.g. Refs. [87–89].

(see e.g. Eqs. (3.60)–(3.62) in Ref. [90]),

$$\left(\delta Z^{[1]}_{\mathcal{H}_2} =\right) \delta Z^{\delta m_t^{\rm OS}}_{\mathcal{H}_2}\Big|_{\rm div} = \delta Z^{\delta m_t^{\rm FIN}}_{\mathcal{H}_2} = \delta Z^{\delta m_t^{\overline{\rm DR}}}_{\mathcal{H}_2} \left(= \delta Z^{[2]}_{\mathcal{H}_2}\right), \tag{28}$$

provided that also in $\delta Z^{\delta m_t^{\rm OS}}_{\mathcal{H}_2}$ all finite pieces are dropped, as done in Ref. [1]. Differences between $\delta Z^{[1]}_{\mathcal{H}_2}$ and $\delta Z^{[2]}_{\mathcal{H}_2}$ arise only at the three-loop level. Consequently, the claim in Ref. [2] that using $\delta m_t^{\rm OS}$ leads to an inconsistency in the Higgs field renormalization in Ref. [1] is not correct. The field renormalizations thus cannot be responsible for the observed differences between Refs. [1,2].

More explicitly, the difference between the two calculations results from non-vanishing δm_t^{ε} terms in the renormalized Higgs-boson self-energies. Those terms naturally appear when performing a full expansion in the dimensional regulator ε. The latter corresponds to choosing $\delta m_t^{\rm OS}$ (as done in Ref. [1]) instead of $\delta m_t^{\rm FIN}$ (as done in Ref. [2]).

In order to isolate the contributions coming from $\mathcal{O}(\varepsilon)$ terms $\times$ $1/\varepsilon$ poles we define the following quantities, where superscripts OS, FIN refer to the respective use of $\delta m_t^{\rm OS}$, $\delta m_t^{\rm FIN}$:

$$\delta T_i^{(2)\rm OS} = \delta T_i^{(2)\,\rm FIN} + \delta_{T_i}\ , \tag{29a}$$
$$\Sigma^{(2)\rm OS}_{\phi_{ij}}(p^2) = \Sigma^{(2)\,\rm FIN}_{\phi_{ij}}(p^2) + \delta_{\Sigma_{ij}}(p^2)\ , \tag{29b}$$
$$\Sigma^{(2)\rm OS}_{AA}(p^2) = \Sigma^{(2)\,\rm FIN}_{AA}(p^2) + \delta_A(p^2)\ , \tag{29c}$$

where the last equation yields a shift for the A-boson mass counterterm in Eq. (7),

$$\delta M_A^{2(2)\rm OS} = \delta M_A^{2(2)\,\rm FIN} + \delta_A(M_A^2). \tag{30}$$

The δ-terms are defined as the *finite* contributions stemming from δm_t^{ε}-dependent parts in the counterterms (see the left diagram in Fig. 2 for an example). The $\overline{\rm DR}$-renormalized quantities do not contain a finite δm_t^{ε}-dependent part by definition. Furthermore, since ϕ_1 has no coupling to the top quark, there are no terms proportional to δm_t^{ε} in $\Sigma^{(2)}_{\phi_1}$, $\Sigma^{(2)}_{\phi_1\phi_2}$, and $\delta T_1^{(2)}$, and it is sufficient to consider $\delta_{\Sigma_{22}}$, δ_A, and δ_{T_2} only. While δ_{T_2} is p^2-independent, we find

$$\delta_{\Sigma_{22}}(p^2) = \frac{3\alpha_t}{2\pi}\, p^2 \frac{\delta m_t^{\varepsilon}}{m_t} + \delta_{\Sigma_{22}}(0), \tag{31}$$
$$\delta_A(p^2) = \frac{3\alpha_t}{2\pi}\, p^2 \cos^2\beta\, \frac{\delta m_t^{\varepsilon}}{m_t} + \delta_A(0). \tag{32}$$

Using Eqs. (3), (11) we find that the following relations hold for the renormalized Higgs-boson self-energies:

$$-\sin^2\beta\,\delta_A(0) - \frac{e}{2M_W s_{\rm w}}\cos^2\beta\sin\beta\,\delta_{T_2} = 0 \quad (\text{for } \hat\Sigma^{(2)}_{\phi_1}),$$
$$\sin\beta\cos\beta\,\delta_A(0) + \frac{e}{2M_W s_{\rm w}}\cos^3\beta\,\delta_{T_2} = 0 \quad (\text{for } \hat\Sigma^{(2)}_{\phi_1\phi_2}),$$
$$\delta_{\Sigma_{22}}(0) - \cos^2\beta\,\delta_A(0) + \frac{e}{2M_W s_{\rm w}}\sin\beta(1+\cos^2\beta)\,\delta_{T_2} = 0 \quad (\text{for } \hat\Sigma^{(2)}_{\phi_2}). \tag{33}$$

This is in agreement with the observation that in the renormalized Higgs-boson self-energies at zero external momentum at $\mathcal{O}(\alpha_t\alpha_s)$, the terms containing δm_t^{ε} drop out in the final (finite) result. Such a cancellation is to be expected as the same combination of one-loop self-energies that potentially contributes to this finite contribution also appears in the $\mathcal{O}(1/\varepsilon)$ term, where they must cancel. This argument in principle still holds when the momentum-dependent $\mathcal{O}(\alpha_t\alpha_s)$ corrections are calculated and *all* counterterms are evaluated with a full expansion in ε. Since the counterterm δ_A is evaluated at $p^2 = M_A^2$, and the Higgs-boson fields are renormalized in the $\overline{\rm DR}$ scheme, however, one finds, using Eqs. (3) and (11) for the three renormalized Higgs-boson self-energies,

$$-\sin^2\beta\left(\delta_A(M_A^2) - \delta_A(0)\right) = \frac{3\alpha_t}{2\pi}\left(-\cos^2\beta\,\sin^2\beta\,M_A^2\right)\frac{\delta m_t^{\varepsilon}}{m_t} \quad \left(\text{for } \hat\Sigma^{(2)}_{\phi_1}\right),$$
$$\sin\beta\cos\beta\left(\delta_A(M_A^2) - \delta_A(0)\right) = \frac{3\alpha_t}{2\pi}\left(+\cos^3\beta\,\sin\beta\,M_A^2\right)\frac{\delta m_t^{\varepsilon}}{m_t} \quad \left(\text{for } \hat\Sigma^{(2)}_{\phi_1\phi_2}\right),$$
$$\left(\delta_{\Sigma_{22}}(p^2) - \delta_{\Sigma_{22}}(0)\right) - \cos^2\beta\left(\delta_A(M_A^2) - \delta_A(0)\right) = \frac{3\alpha_t}{2\pi}\left(p^2 - \cos^4\beta\,M_A^2\right)\frac{\delta m_t^{\varepsilon}}{m_t} \quad \left(\text{for } \hat\Sigma^{(2)}_{\phi_2}\right), \tag{34}$$

i.e. the δm_t^{ε} terms contribute in the newly evaluated $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections. They are p^2-independent in $\hat\Sigma^{(2)}_{\phi_1}$ and $\hat\Sigma^{(2)}_{\phi_1\phi_2}$, while they do depend on p^2 in $\hat\Sigma^{(2)}_{\phi_2}$.

The p^2-dependent terms coming from the expansion of terms like $(-p^2)^{-\varepsilon}$ multiplying a $1/\varepsilon^2$ divergence must certainly cancel after inclusion of the counterterms, because non-local terms cannot appear in a renormalizable theory. However, the cancellation of the ε-dependent terms stemming from the mass renormalization is not necessarily fulfilled once the two-loop amplitude carries full momentum dependence. Similarly, the truncation of the field renormalization to the divergent part cuts away terms involving δm_t^{ε}, leading to further non-cancellations. The explicit $\overline{\rm DR}$ renormalization of the Higgs-boson fields drops the corresponding finite contributions, such that no $\delta m_t^{\rm fin}$, δm_t^{ε} terms are taken into account. The different dependence on the external momentum and the $\overline{\rm DR}$ prescription for the Higgs field renormalization leads to Eqs. (34).

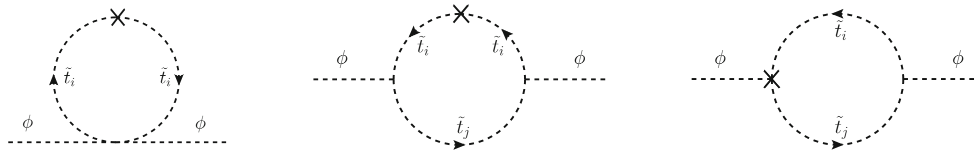

Fig. 3 One-loop subrenormalization diagrams containing top-squark loops with counterterm insertions

Equivalent momentum-dependent terms of $\mathcal{O}(\varepsilon)$ of the scalar-top mass counterterms, evaluated from the diagrams in the lower row of Fig. 1, do not contribute. The diagrams with top-squark counterterm insertions are depicted in Fig. 3. The first diagram is momentum independent. In the second diagram, the corresponding loop integral is a massive scalar three-point function (C_0) with only scalar particles running in the loop, and thus is UV finite. Consequently, the top-squark mass counterterm insertions of $\mathcal{O}(\varepsilon)$ do not contribute. In the third diagram the stop mass counterterm can enter via the (dependent) counterterm for A_t [29,84]. This diagram does not possess a momentum-dependent divergence, however, and thus the $\mathcal{O}(\varepsilon)$ term of the scalar-top mass counterterm again does not contribute.

3.2 Physics content and interpretation

In the following we give another view on the finite δm_t^ε term from the top mass renormalization and on the interpretation of the different results for the Higgs-boson masses with and without this term.

In the approximation with $p^2 = 0$ for the two-loop self-energies, the results are the same for either dropping or including the δm_t^ε term, provided that this is done everywhere in the contributions from the top–stop sector in the renormalized two-loop self-energies.

As explained above, abandoning the $p^2 = 0$ approximation yields an additional δm_t^ε in the p^2-coefficient of the self-energy $\Sigma^{(2)}_{\phi_2}(p^2)$ when the on-shell top-quark mass counterterm, see Eq. (18), is used, as well as in the A-boson self-energy $\Sigma_{AA}(p^2)$ from which it induces an additive term $\sim M_A^2\, \delta m_t^\varepsilon / m_t$ to the mass counterterm δM_A^2.

In the renormalized self-energy $\hat{\Sigma}^{(2)}_{\phi_2}(p^2)$, Eq. (3c), this extra p^2-dependent term survives when $\delta Z^{(2)}_{\mathcal{H}_2}$ is defined in the minimal way containing only the $1/\varepsilon$ and $1/\varepsilon^2$ singular parts; however, it disappears in $\hat{\Sigma}^{(2)}_{\phi_2}(p^2)$ when the minimal $\delta Z^{(2)}_{\mathcal{H}_2} = \delta Z^{\delta m_t^{\rm OS}\,(2)}_{\mathcal{H}_2}\Big|_{\rm div}$ is replaced by

$$\delta Z^{(2)}_{\mathcal{H}_2} \to \delta Z^{(2)}_{\mathcal{H}_2} - \frac{3\alpha_t}{2\pi}\frac{\delta m_t^\varepsilon}{m_t}, \tag{35}$$

which now accommodates also a finite part of two-loop order.

This shift in $\delta Z^{(2)}_{\mathcal{H}_2}$ by a finite term has also an impact on the counterterm for $\tan\beta$ via $\delta\tan\beta = \frac{1}{2}\delta Z^{(2)}_{\mathcal{H}_2}$. This has the consequence that the extra δm_t^ε term in δM_A^2 drops out in the constant counterterms for the renormalized self-energies $\hat{\Sigma}^{(2)}_{\phi_{ij}}(p^2)$ in Eq. (3) because of cancellations with the δm_t^ε term in $\delta\tan\beta$ and $\delta Z^{(2)}_{\mathcal{H}_2}$ [this can be seen from the explicit expressions given in Eqs. (6) and (11)].

Accordingly, keeping or dropping the finite δm_t^ε part is thus equivalent to a finite shift in the field-renormalization constant $\delta Z_{\mathcal{H}_2}$ at the two-loop level, which corresponds to a finite shift in $\tan\beta$ as input quantity. Numerically, the shift in $\tan\beta$ is small, and cannot explain the differences in the M_h predictions from the two schemes. Hence, these differences originate from the different p^2 coefficients in $\hat{\Sigma}^{(2)}_{\phi_2}(p^2)$.

The impact of a modification of the two-loop field-renormalization constant on the mass M_h can best be studied in terms of the self-energy Σ_{hh} in the h, H basis, which is composed of the $\Sigma_{\phi_{ij}}$ in the following way:

$$\Sigma_{hh} = \cos^2\alpha\, \Sigma_{\phi_2} + \sin^2\alpha\, \Sigma_{\phi_1} - 2\sin\alpha\cos\alpha\, \Sigma_{\phi_1\phi_2}, \tag{36}$$

where only Σ_{ϕ_2} contains the p^2-dependent δm_t^ε contribution. In order to simplify the discussion and to point to the main features, we assume sufficiently large values of $\tan\beta$, such that we can write $\hat{\Sigma}_{hh} \simeq \hat{\Sigma}_{\phi_2}$, and h, H mixing effects play only a marginal role (both simplifications apply to the numerical discussions in the subsequent section). Moreover, to simplify the notation, we drop the indices and define

$$\Sigma_{hh} \equiv \Sigma, \quad \hat{\Sigma}_{hh} \equiv \hat{\Sigma}, \quad \delta Z_{hh} \equiv \delta Z, \tag{37}$$

where $\delta Z_{hh} = \cos^2\alpha\, \delta Z_{\mathcal{H}_2} + \sin^2\alpha\, \delta Z_{\mathcal{H}_1} \simeq \delta Z_{\mathcal{H}_2}$. Starting from the tree-level mass m_h and the renormalized h self-energy up to the two-loop level,

$$\hat{\Sigma}(p^2) = \Sigma(p^2) - \delta m_h^2 + \delta Z(p^2 - m_h^2), \tag{38}$$

we obtain the higher-order corrected mass M_h from the pole of the propagator, i.e.

$$M_h^2 - m_h^2 + \hat{\Sigma}(M_h^2) = 0. \tag{39}$$

The Taylor-expansion of the unrenormalized self-energy around $p^2 = 0$,

$$\Sigma(p^2) = \Sigma(0) + p^2\, \Sigma'(0) + \tilde{\Sigma}(p^2), \tag{40}$$

yields the first two terms containing the singularities in $1/\varepsilon$ and $1/\varepsilon^2$, and the residual fully finite and scheme-independent part denoted by $\tilde{\Sigma}(p^2)$. With this expansion inserted into Eq. (38) one obtains from the pole condition Eq. (39) the relation

$$(M_h^2 - m_h^2)\left[1+\delta Z+\Sigma'(0)\right] + \left[\Sigma(0) - \delta m_h^2 + m_h^2\,\Sigma'(0)\right] + \tilde{\Sigma}(M_h^2) = 0, \tag{41}$$

where the expressions in the square brackets are each finite, irrespective of a possible finite term in the definition of δZ.

Taking into account that M_h^2 differs from m_h^2 by a higher-order shift, we can replace

$$\tilde{\Sigma}(M_h^2) = \tilde{\Sigma}(m_h^2) + (M_h^2 - m_h^2)\,\tilde{\Sigma}'(m_h^2) + \cdots \tag{42}$$

and obtain

$$\begin{aligned} M_h^2 - m_h^2 &= -\frac{\Sigma(0) - \delta m_h^2 + m_h^2\,\Sigma'(0) + \tilde{\Sigma}(m_h^2)}{1+\delta Z + \Sigma'(0) + \tilde{\Sigma}'(m_h^2)} \\ &= -\left[\Sigma(0) - \delta m_h^2 + m_h^2\,\Sigma'(0) + \tilde{\Sigma}(m_h^2)\right]_{1\mathrm{loop}+2\mathrm{loop}} \\ &\quad + \left[\Sigma(0) - \delta m_h^2 + m_h^2\,\Sigma'(0) + \tilde{\Sigma}(m_h^2)\right]_{1\mathrm{loop}} \\ &\quad \cdot \left[\delta Z + \Sigma'(0) + \tilde{\Sigma}'(m_h^2)\right]_{1\mathrm{loop}} + \cdots \end{aligned} \tag{43}$$

showing explicitly all terms up to two-loop order. It does not contain the two-loop part of the field-renormalization constant, which indeed would show up at the three-loop level. Hence, effects resulting from different conventions for $\delta Z^{(2\mathrm{loop})}$ in the finite part have to be considered in the current situation as part of the theoretical uncertainty.

3.3 Numerical comparison

In this section the renormalized momentum-dependent $\mathcal{O}(p^2\alpha_t\alpha_s)$ self-energy contributions $\Delta\hat{\Sigma}_{hh}, \Delta\hat{\Sigma}_{hH}, \Delta\hat{\Sigma}_{HH}$ of Eq. (15) and the mass shifts

$$\Delta M_h = M_h - M_{h,0}, \quad \Delta M_H = M_H - M_{H,0} \tag{44}$$

are compared using either δm_t^{OS} or $\delta m_t^{\mathrm{FIN}}$, as discussed above. $M_{h,0}$ and $M_{H,0}$ denote the Higgs-boson mass predictions *without* the newly obtained $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections.

The results are obtained for two different scenarios. Scenario 1 is adopted from the $m_h^{\max}$ scenario described in Ref. [91]. We use the following parameters:

$$m_t = 173.2~\text{GeV}, \quad M_{\mathrm{SUSY}} = 1~\text{TeV}, \quad X_t = 2\,M_{\mathrm{SUSY}},$$
$$m_{\tilde{g}} = 1500~\text{GeV}, \quad \mu = M_2 = 200~\text{GeV}. \tag{45}$$

Here M_2 denotes the $SU(2)$ soft SUSY-breaking parameter, where the $U(1)$ parameter is derived via the GUT relation $M_1 = (5/3)\,(s_\mathrm{w}^2/c_\mathrm{w}^2)\,M_2$. Scenario 2 is an updated version of the "light-stop scenario" of Refs. [91,92]

$$m_t = 173.2~\text{GeV}, \quad M_{\mathrm{SUSY}} = 0.5~\text{TeV}, \quad X_t = 2\,M_{\mathrm{SUSY}},$$
$$m_{\tilde{g}} = 1500~\text{GeV}, \quad \mu = M_2 = 400~\text{GeV}\; M_1 = 340~\text{GeV}, \tag{46}$$

leading to stop mass values of

$$m_{\tilde{t}_1} = 326.8~\text{GeV}, \quad m_{\tilde{t}_2} = 673.2~\text{GeV}. \tag{47}$$

A renormalization scale of $\mu = m_t$ is set in all numerical evaluations.

Self-energies

In Fig. 4 we present the results for the δ_A (upper plot) and $\delta_{\Sigma_{22}}$ (lower plot) contributions for $\tan\beta = 5(20)$ in red (blue) in Scenario 1, where δ_A, $\delta_{\Sigma_{22}}$ are defined in Eqs. (31) and (32). In the upper plot $\delta_A(M_A^2)$ ($\delta_A(0)$) is shown as solid (dashed) line; correspondingly, in the lower plot $\delta_{\Sigma_{22}}(p^2)$ ($\delta_{\Sigma_{22}}(0)$) is depicted as solid (dashed) line. The contribution is seen to decrease quadratically with M_A or p ($:= \sqrt{p^2}$) when including the momentum-dependent terms; see Eq. (34). For δ_A it is suppressed with $\tan^2\beta$. For high values of M_A and low $\tan\beta$, the δ_A contribution becomes sizable. Similarly, for large p the $\delta_{\Sigma_{22}}$ term becomes sizable, showing the relevance of the δm_t^{ε} contribution.

The behavior of the real parts of the two-loop contributions to the self-energies $\Delta\hat{\Sigma}_{ab}$ is analyzed in Fig. 5. Solid lines show the result evaluated with δm_t^{OS}, as obtained in Ref. [1] [i.e. the new contribution added to the previous `FeynHiggs` result in Ref. [1]; see Eq. (15)]. Dashed lines show the result evaluated with $\delta m_t^{\mathrm{FIN}}$, as obtained in Ref. [2]. We show $M_A = 250$ GeV and $\tan\beta = 5(20)$ as red (blue) lines. The difference between the $\delta m_t^{\mathrm{FIN}}$ and δm_t^{OS} calculations for $\Delta\hat{\Sigma}_{\phi_1}$ and $\Delta\hat{\Sigma}_{\phi_1\phi_2}$ is p-independent, as discussed below Eq. (34), and the difference between the two schemes is numerically small. For $\Delta\hat{\Sigma}_{\phi_2}$, on the other hand, the difference becomes large for large values of p. This self-energy contribution is mostly relevant for the light $\mathcal{CP}$-even Higgs boson, however, i.e. for $p \sim M_h$, and thus the *relevant* numerical difference remains relatively small (but non-zero) compared to the larger differences at large p.

For completeness it should be mentioned that the imaginary part is not affected by the variation of the top-quark renormalization, as only the real parts of the counterterm insertions enter the calculation.

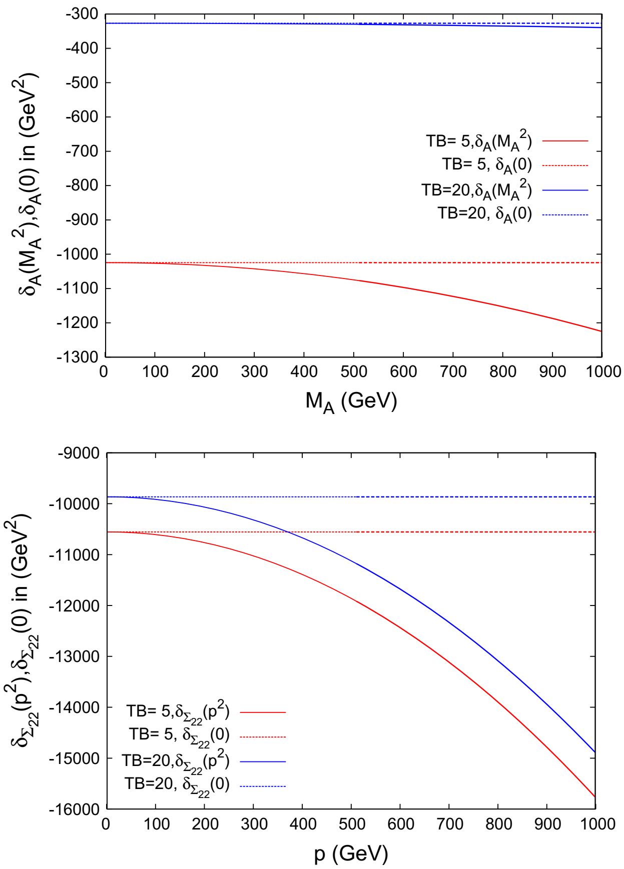

Fig. 4 $\delta_A(M_A^2)$ and $\delta_A(0)$ varying M_A shown in the *upper plot*, $\delta_{\Sigma_{22}}(p^2)$ and $\delta_{\Sigma_{22}}(0)$ in the *lower plot*, both within Scenario 1

Scenario 2 was omitted as the relevant aspects for the analysis of the self-energies using δm_t^{OS} vs. δm_t^{FIN} have become sufficiently apparent within Scenario 1.

Mass shifts

We now turn to the effects on the neutral $\mathcal{CP}$-even Higgs-boson masses themselves. The numerical effects on the two-loop corrections to the Higgs-boson masses $M_{h,H}$ are investigated by analyzing the mass shifts ΔM_h and ΔM_H of Eq. (44). The results are shown for the two renormalization schemes for the top-quark mass, i.e. using δm_t^{OS} or δm_t^{FIN}. The color coding is as in Fig. 5. The results for Scenario 1 are shown in Fig. 6 and are in agreement with Figs. 2 and 3 (left) in Ref. [2], i.e. we reproduce the results of Ref. [2] using δm_t^{FIN}.

The results for Scenario 2 are shown in Fig. 7. The results are again in agreement with Figs. 2 and 3 (right) in Ref. [2]. This agreement confirms the use of δm_t^{FIN} in Ref. [2], in comparison with δm_t^{OS} used in the evaluation of our results.

For the contribution to M_H, peaks can be observed at $M_A = 2m_{\tilde{t}_1}$, $m_{\tilde{t}_1} + m_{\tilde{t}_2}$, $2m_{\tilde{t}_2}$; see also Ref. [1] and the discussion of Fig. 9 below.

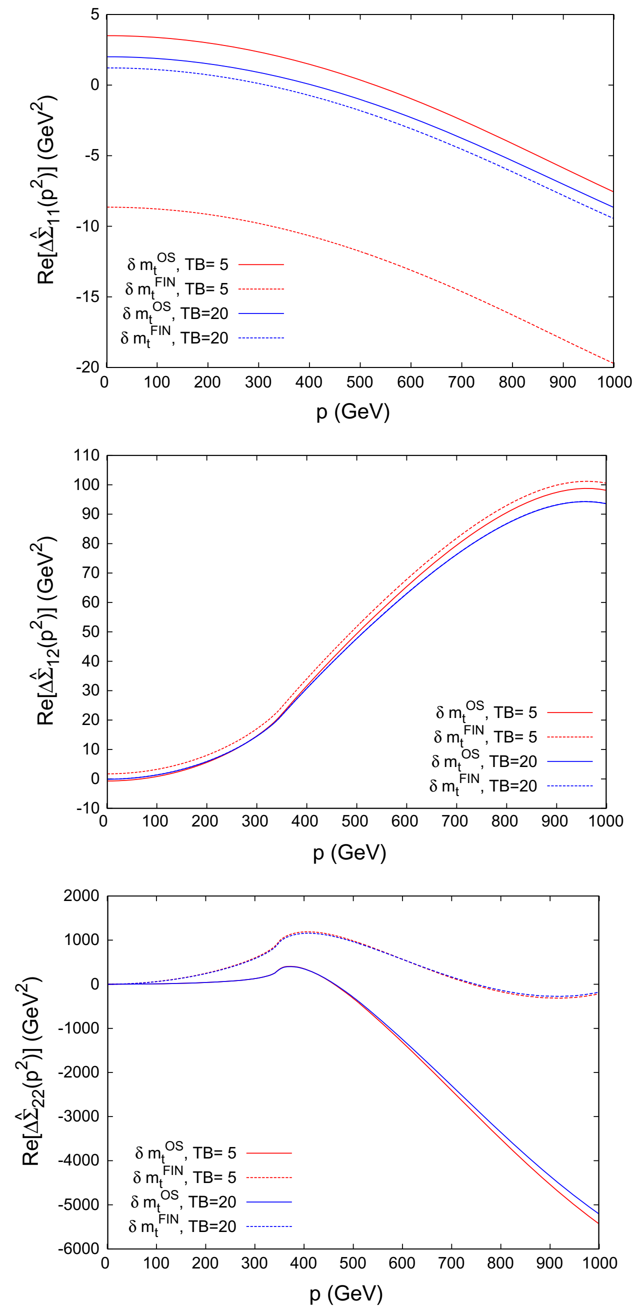

Fig. 5 $\Delta\hat{\Sigma}_{\phi_{ij}}$ in Scenario 1 (with $M_A = 250$ GeV) for $ij = 11, 12, 22$ in the *upper*, the *middle* and the *lower plot*, respectively. The *solid* (*dashed*) *lines* show the result obtained with δm_t^{OS} (δm_t^{FIN}); the *red* (*blue*) *lines* correspond to $\tan\beta = 5(20)$

Since the results using δm_t^{OS} and δm_t^{FIN} correspond to two different renormalization schemes, their difference should be regarded as an indication of missing higher-order momentum-dependent corrections.

4 Comparison with the m_t $\overline{\textbf{DR}}$ renormalization

Having examined the renormalization of the top-quark mass, we will now analyze the numerical differences between an

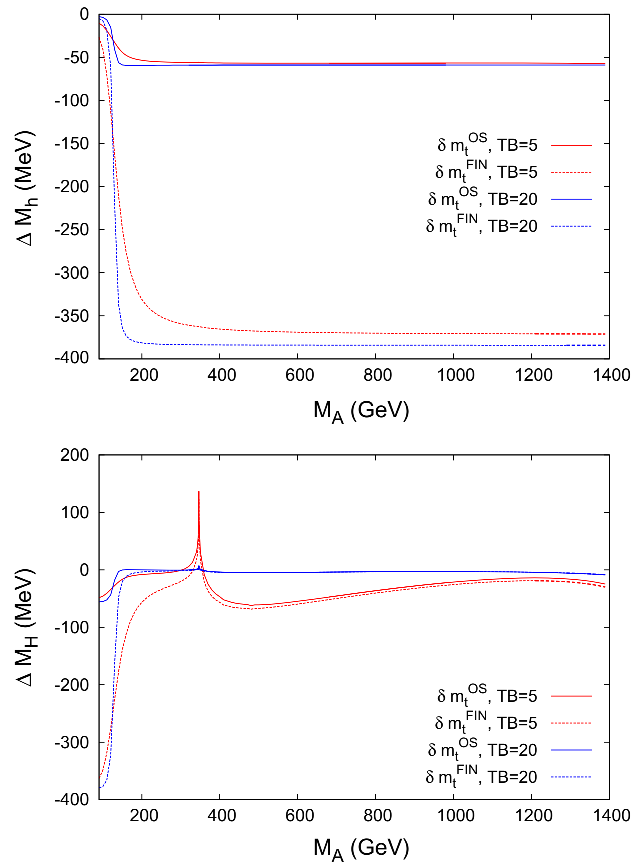

Fig. 6 Variation of the mass shifts ΔM_h, ΔM_H with the A-boson mass M_A within Scenario 1, for $\tan\beta = 5$ (*red*) and $\tan\beta = 20$ (*blue*) including or excluding some δ terms. The peak in ΔM_H originates from a threshold at $2\,m_t$

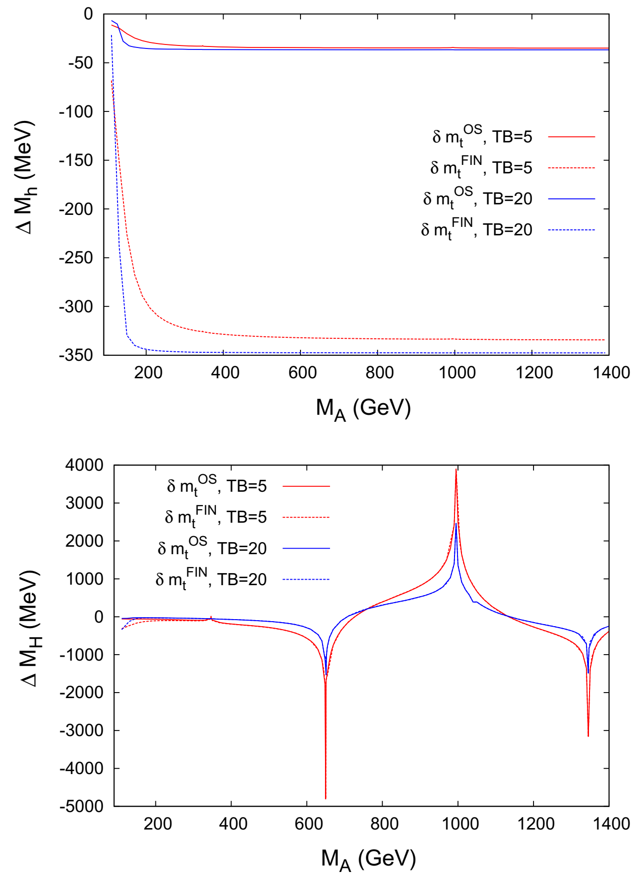

Fig. 7 Variation of the mass shifts ΔM_h, ΔM_H with the A-boson mass M_A within Scenario 2, for $\tan\beta = 5$ (*red*) and $\tan\beta = 20$ (*blue*) including or excluding some δ terms. The peaks in ΔM_H originate from thresholds at $2\,m_t$, $2\,m_{\tilde{t}_1}$, $m_{\tilde{t}_1} + m_{\tilde{t}_2}$, and $2\,m_{\tilde{t}_2}$, where the threshold at $2\,m_t$ is suppressed by $1/\tan^2\beta$

$m_t^{\overline{\rm DR}}$ and an $m_t^{\rm OS}$ calculation. This has been realized by employing a $\overline{\rm DR}$ renormalization of the top-quark mass in all steps of the calculation. The top-squark masses are kept renormalized on-shell. This can be seen as an intermediate step toward a full $\overline{\rm DR}$ analysis.

4.1 Implementation in the program FeynHiggs

In the $\overline{\rm DR}$ scheme the top-quark mass parameter entering the calculation is the MSSM $\overline{\rm DR}$ top-quark mass, which at one-loop order is related to the pole mass m_t (given in the user input) in the following way:

$$m_t^{\overline{\rm DR}}(\mu) = m_t \cdot \left[1 + \frac{\delta m_t^{\rm fin}}{m_t} + \mathcal{O}\left(\left(\alpha_s^{\overline{\rm DR}}\right)^2\right)\right]. \tag{48}$$

The term $\delta m_t^{\rm fin}$ can be obtained from Eq. (18), with the formal replacement $\alpha_s \to \alpha_s^{\overline{\rm DR}}(\mu)$, yielding

$$\begin{aligned}\frac{\delta m_t^{\rm fin}}{m_t} = \alpha_s^{\overline{\rm DR}}(\mu) \Bigg(&-\frac{5}{3\pi} + \frac{1}{\pi}\log(m_t^2/\mu^2) \\ &+ \frac{m_{\tilde g}^2}{3m_t^2\pi}\left(-1 + \log(m_{\tilde g}^2/\mu^2)\right) \\ &+ \frac{1}{6m_t^2\pi}\Big(m_{\tilde t_1}^2(1 - \log(m_{\tilde t_1}^2/\mu^2)) \\ &+ m_{\tilde t_2}^2(1 - \log(m_{\tilde t_2}^2/\mu^2)) \\ &+ (m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2 - 2m_{\tilde g}m_t\sin(2\theta_t)) \\ &\times {\rm Re}[B_0^{\rm fin}(m_t^2, m_{\tilde g}^2, m_{\tilde t_1}^2)] \\ &+ (m_{\tilde g}^2 + m_t^2 - m_{\tilde t_2}^2 + 2m_{\tilde g}m_t\sin(2\theta_t)) \\ &\times {\rm Re}[B_0^{\rm fin}(m_t^2, m_{\tilde g}^2, m_{\tilde t_2}^2)]\Big)\Bigg).\end{aligned} \tag{49}$$

At zeroth order, $\alpha_s^{\overline{\rm DR}}(\mu) = \alpha_s^{\overline{\rm MS}}(\mu)$.

As on-shell renormalized quantities the stop masses $m_{\tilde t_1}$ and $m_{\tilde t_2}$ should have fixed values, independently of the renormalization chosen for the top-quark mass. We compensate

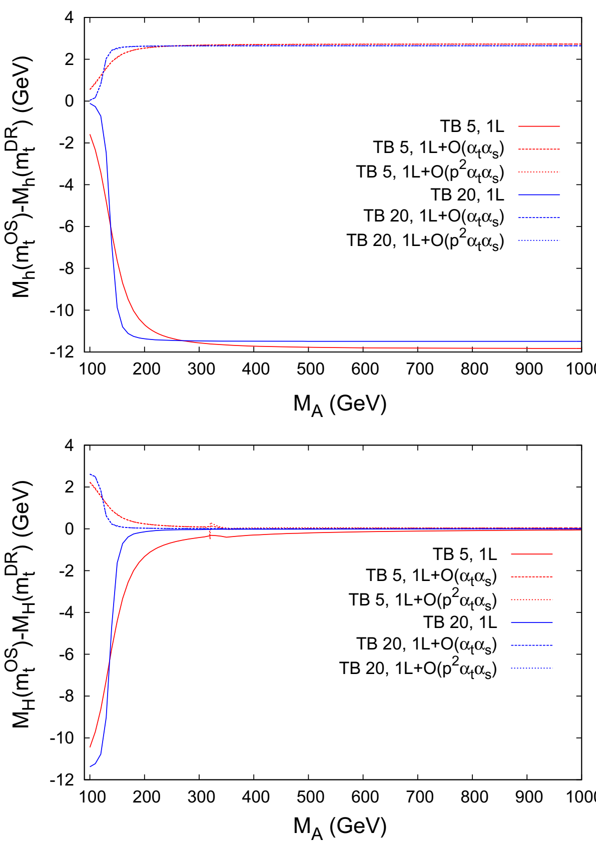

Fig. 8 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*). The difference is shown as *solid* (*dashed/dotted*) *line* at the one-loop ($\mathcal{O}(\alpha_t\alpha_s)/\mathcal{O}(p^2\alpha_t\alpha_s)$) level as a function of M_A for $\tan\beta = 5(20)$ in *red* (*blue*) within Scenario 1

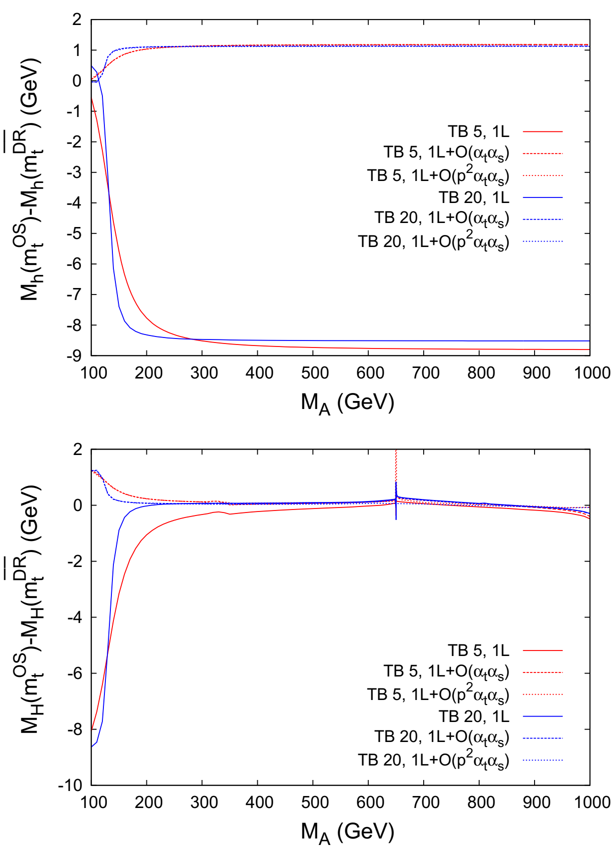

Fig. 9 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*) as a function of M_A within Scenario 2, with the same *line/color* coding as in Fig. 8. The peak in the *lower plot* originates from a threshold at $2\, m_{\tilde{t}_1}$. The threshold at $2\, m_t$ is suppressed by $1/\tan^2\beta$

for the changes induced by $\delta m_t^{\rm fin}$ in the stop mass matrix, Eq. (13), by shifting the SUSY-breaking parameters as follows:

$$M_{\tilde{t}_L}^2 \to M_{\tilde{t}_L}'^2 = M_{\tilde{t}_L}^2 + (m_t^{\rm OS})^2 - (m_t^{\overline{\rm DR}})^2, \tag{50a}$$

$$7M_{\tilde{t}_R}^2 \to M_{\tilde{t}_R}'^2 = M_{\tilde{t}_R}^2 + (m_t^{\rm OS})^2 - (m_t^{\overline{\rm DR}})^2, \tag{50b}$$

$$A_t \to A_t' = \frac{m_t^{\rm OS}}{m_t^{\overline{\rm DR}}}\left(A_t - \frac{\mu}{\tan\beta}\right) + \frac{\mu}{\tan\beta}. \tag{50c}$$

(Except for A_t, which actually appears in the Feynman rules, `FeynHiggs` only pretends to perform these shifts but computes the sfermion masses using $m_t^{\rm OS}$.)

This procedure is available in `FeynHiggs` from version 2.11.1 on and is activated by setting the new value 2 for the `runningMT` flag. The comparison of the results with $\overline{\rm DR}$ and with OS renormalization admits an improved estimate of (some) of the missing three-loop corrections in the top/stop sector.

4.2 Numerical analysis

In the following plots we show the difference

$$\bar{\Delta} M_\phi := M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}}), \quad \phi = h, H, \tag{51}$$

between M_ϕ evaluated in the OS scheme, i.e. using $m_t^{\rm OS}$ (*not* $m_t^{\rm FIN}$), and in the $\overline{\rm DR}$ scheme, i.e. using $m_t^{\overline{\rm DR}}$.

Dependence on M_A

In the upper half of Fig. 8, $\bar{\Delta} M_h$ is plotted in Scenario 1 as a function of M_A with $\tan\beta = 5(20)$ in red (blue). The solid (dashed) lines show the difference evaluated at the full one-loop level (including the $\mathcal{O}(\alpha_t\alpha_s)$ corrections). The dotted lines include the newly calculated $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections. For $M_A \gtrsim 200$ GeV one observes large differences of $\mathcal{O}(10$ GeV$)$ at the one-loop level, indicating the size of missing higher-order corrections from the top/stop sector beyond one loop. This difference is strongly reduced at the two-loop level, to about $\sim$3 GeV, now corresponding to missing higher

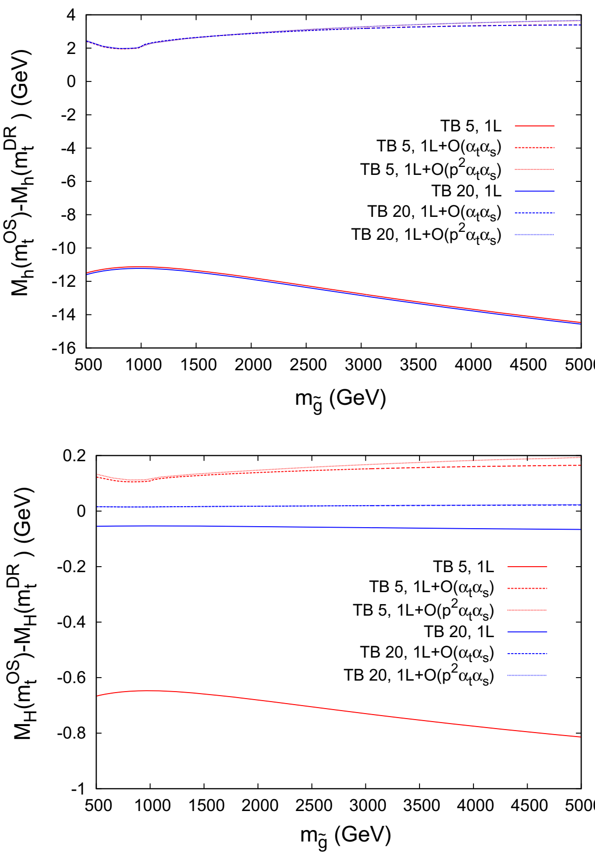

Fig. 10 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*) as a function of $m_{\tilde{g}}$ within Scenario 1, for $M_A =$ 250 GeV and with the same *line/color* coding as in Fig. 8

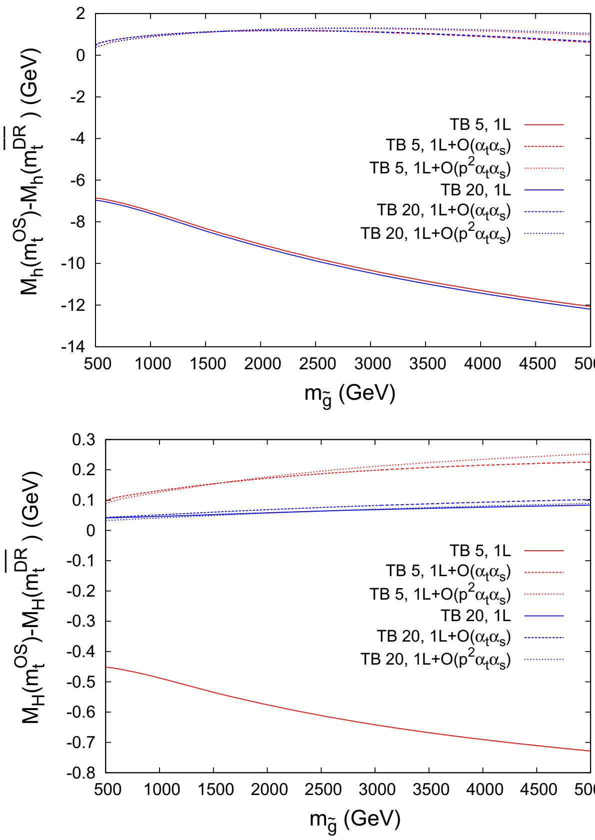

Fig. 11 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*) as a function of $m_{\tilde{g}}$ within Scenario 2, for $M_A =$ 250 GeV and with the same *line/color* coding as in Fig. 8

orders beyond two loops from the top/stop sector. The dotted lines are barely visible below the dashed lines, indicating the relatively small effect of the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections as derived in Ref. [1].

The lower plot of Fig. 8 shows the corresponding results for $\bar{\Delta} M_H$ with the same color/line coding. Here large effects are only visible for low M_A, where the higher-order corrections to M_H are sizable (and the light Higgs boson receives only very small higher-order corrections). In this part of the parameter space the same reduction of $\bar{\Delta} M_H$ going from one loop to two loops can be observed.

The behavior is similar for Scenario 2, shown in Fig. 9 (with the same line/color coding as in Fig. 8), only the size of the difference $\bar{\Delta} M_h$ is $\sim 20\,\%$ smaller at the one-loop level, and $\sim 50\,\%$ smaller at the two-loop level compared to Scenario 1. The same peak structure due to thresholds as in Fig. 7 is visible.

Dependence on $m_{\tilde{g}}$

In Figs. 10 and 11 we analyze $\bar{\Delta} M_\phi$ as a function of $m_{\tilde{g}}$ in Scenario 1 and 2, respectively. We fix $M_A = 250$ GeV and use the same line/color coding as in Fig. 8. Due to the choice of an MSSM $\overline{\rm DR}$ top-quark mass definition, $m_t^{\overline{\rm DR}}$ varies with $m_{\tilde{g}}$ already at the one-loop level.

In the upper plots we show the light $\mathcal{CP}$-even Higgs-boson case, where it can be observed that the scheme dependence is strongly reduced at the two-loop level. It reaches 2–3 GeV in Scenario 1 and $\sim$ 1 GeV in Scenario 2, largely independently of $\tan\beta$. At the one-loop level the scheme dependence grows with $m_{\tilde{g}}$, whereas the dependence is much milder at the two-loop level. The effects of the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections become visible at larger $m_{\tilde{g}}$, in agreement with Ref. [1].

The heavy $\mathcal{CP}$-even Higgs-boson case is shown in the lower plots. At small $\tan\beta$ scheme differences of $\mathcal{O}(600$ MeV$(150$ MeV$))$ can be observed at the one- (two-) loop level. For large $\tan\beta$ the differences always stay below $\mathcal{O}(50$ MeV$)$, in agreement with Fig. 8. The dependence on $m_{\tilde{g}}$ is similar to the light Higgs boson, but again somewhat weaker.

Dependence on X_t

Finally, in Figs. 12 and 13 we analyze $\bar{\Delta} M_\phi$ as a function of $X_t = X_t^{\rm OS}$ in Scenario 1 and 2, respectively. We again

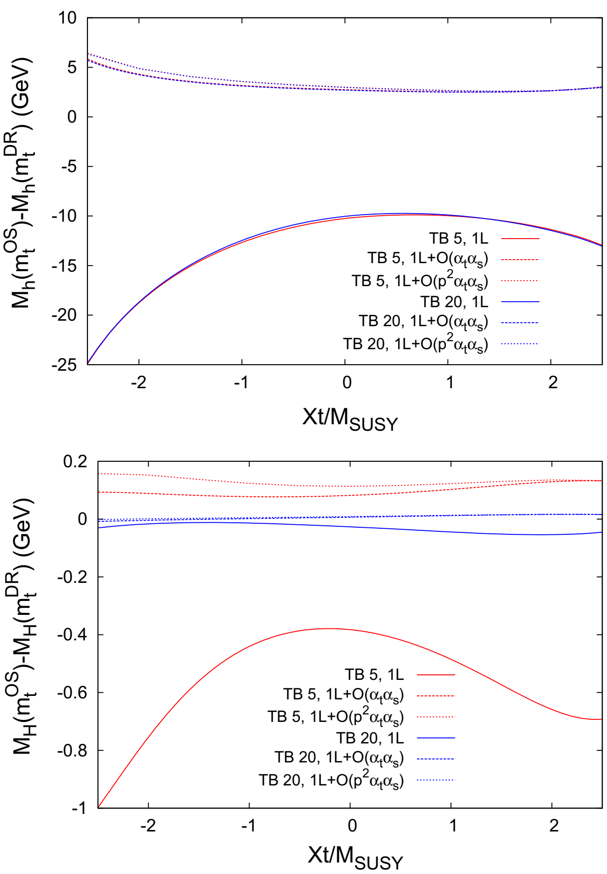

Fig. 12 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*) as a function of $X_t = X_t^{\rm OS}$ within Scenario 1, with the same *line/color* coding as in Fig. 8

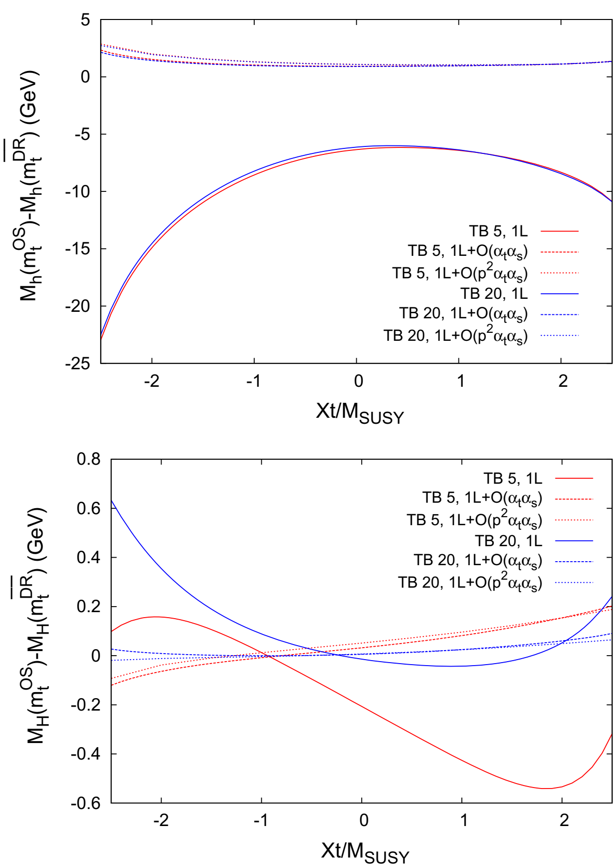

Fig. 13 $\bar{\Delta} M_\phi = M_\phi(m_t^{\rm OS}) - M_\phi(m_t^{\overline{\rm DR}})$ for $\phi = h$ (*upper plot*) and $\phi = H$ (*lower plot*) as a function of $X_t = X_t^{\rm OS}$ within Scenario 2, with the same *line/color* coding as in Fig. 8

fix $M_A = 250$ GeV and use the same line/color coding as in Fig. 8.

In the upper plots we show the light $\mathcal{CP}$-even Higgs-boson case. As before the scheme dependence is strongly reduced when going from the one-loop to the two-loop case. In general a smaller scheme dependence is found from small X_t, while it increases for larger $|X_t|$ values, in agreement with Ref. [93]. For most parts of the parameter space, when the two-loop corrections are included, it is found to be below $\sim$3 GeV. The contribution of $\mathcal{O}(p^2\alpha_t\alpha_s)$ remains small for all X_t values.

In the heavy $\mathcal{CP}$-even Higgs-boson case, shown in the lower plots, the dependence of the size of the effects is slightly more involved, though the general picture of a strongly reduced scheme dependence can be observed here, too. In both scenarios, for large negative X_t and $\tan\beta = 5$ the $\mathcal{O}(p^2\alpha_t\alpha_s)$ contributions can become sizable with respect to the $\mathcal{O}(\alpha_t\alpha_s)$ corrections.

In conclusion, the scheme dependence is found to be reduced substantially when going from the pure one-loop calculation to the two-loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections. This indicates that corrections at the three-loop level and beyond, stemming from the top/stop sector are expected at the order of the observed scheme dependence, i.e. at the level of $\sim$3 GeV. This is in agreement with existing calculations beyond two loops [54–56,60].

A further reduction of the scheme dependence might be expected by adding the $\mathcal{O}(\alpha_t^2)$ contributions. The $m_t^{\overline{\rm DR}}$ value calculated at $\mathcal{O}(\alpha_s+\alpha_t)$ is substantially closer to $m_t^{\rm OS}$, reducing already strongly the scheme dependence at the one-loop level. This extended analysis is beyond the scope of our paper, however.

5 Conclusions

In this paper we analyzed the scheme dependence of the $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the neutral $\mathcal{CP}$-even Higgs-boson masses in the MSSM. In a first step we investigated the differences in the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections as obtained in Refs. [1,2]. We have shown that the difference can be attributed to different renormalizations of the top-quark mass. In both calculations an "on-shell" top-quark mass was employed. The evaluation in Ref. [1] includes the $\mathcal{O}(\varepsilon)$ terms of the top-quark mass counterterm, δm_t^ε, however, whereas this contribution was omitted in Ref. [2]. We have shown analytically that the terms involving δm_t^ε do *not* cancel in the $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections to the renormalized Higgs-boson self-energies (an effect that was already observed in the $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the NMSSM Higgs sector [90]). Numerical agreement between Refs. [1,2] is found as soon as the δm_t^ε terms are dropped from the calculation in Ref. [1]. Moreover, as an alternative interpretation, we have shown that omitting the δm_t^ε terms is equivalent to a redefinition of the finite part of the two-loop field-renormalization constant which affects the Higgs-boson mass prediction at the three-loop order (apart from a numerically insignificant shift in $\tan\beta$ as an input parameter). The differences between the two calculations can thus be regarded as an indication of the size of the missing momentum-dependent corrections beyond the two-loop level, and they reach up to several hundred MeV in the case of the light $\mathcal{CP}$-even Higgs boson.

In a second step we performed a calculation of the $\mathcal{O}(\alpha_t\alpha_s)$ and $\mathcal{O}(p^2\alpha_t\alpha_s)$ corrections employing a $\overline{\mathrm{DR}}$ top-quark mass counterterm. We analyzed the numerical difference of the Higgs-boson masses evaluated with δm_t^{OS} and with $\delta m_t^{\overline{\mathrm{DR}}}$. By varying the $\mathcal{CP}$-odd Higgs-boson mass, M_A, the gluino mass, $m_{\tilde{g}}$ and the off-diagonal entry in the scalar-top mass matrix, X_t, we found that in all cases the scheme dependence, in particular of the light $\mathcal{CP}$-even Higgs-boson mass, is strongly reduced by going from the full one-loop result to the two-loop result including the $\mathcal{O}(\alpha_t\alpha_s)$ corrections. The further inclusion of the $\mathcal{O}(p^2\alpha_t\alpha_s)$ contributions had a numerically small effect. The differences found at the two-loop level indicate that corrections at the three-loop level and beyond, stemming from the top/stop sector, are expected at the level of $\sim$3 GeV. This is in agreement with existing calculations beyond two loops [54–56,60]. The possibility to use $m_t^{\overline{\mathrm{DR}}}$ instead of m_t^{OS} has been added to the `FeynHiggs` package and allows an improved estimate of the size of missing corrections beyond the two-loop order.

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